

# A SURVEY OF MATHEMATICAL TECHNIQUES IN FINGER VEIN BIOMETRICS

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**Abstract**—This paper presents a survey of some of the important mathematical techniques used in finger vein biometrics and comparison of those techniques used in each stages of finger vein recognition. Much of the existing methods are based on mathematical fundamentals in conjunction with image processing techniques. This also briefly discusses each image processing step and existing techniques in each step associated with finger vein biometrics. We will show how mathematics is related to some of the major problems in finger vein biometrics including image enhancement, feature extraction and matching. The paper concludes with directions for further research.

**Keywords:** Finger Vein Biometrics, Image Processing, Image Enhancement, Feature Extraction and Matching

## INTRODUCTION

Biometrics has been widely adopted for secure authentication because of its advantage over other conventional methods like password based or card based authentication. The fingerprint, face, iris are some of the common biometric solutions. But, all of them have some or other limitations and the search for an ideal biometric mechanism continues. Finger vein biometric is a new technology which authenticates a person using the vein patterns in his finger. Unlike fingerprint, vein patterns are hidden inside the skin ensuring more security.

Mathematical models are the foundation of image processing. Based on those models in image processing the vein image collected from a person is enhanced, features are extracted and matched for verification. A key research area is the formulation of algorithms and methods based on mathematical techniques in order to develop a secure and reliable finger vein biometric system. Specifically, in finger vein recognition we have four key processes:

1. Image Acquisition;
2. Pre-processing;
3. Feature extraction;
4. Pattern matching.

In this paper, we consider all the processes except the first one since first process is simply collecting the vein image of the user. All other processes apply some mathematical methods to the input data and processed data is passed on to next process as input. There exist a number of techniques for each of these steps. Here, such methods are briefly discussed and compared.

## PREPROCESSING

Vein images typically suffer from one or more of the following imperfections:

1. Low contrast;
2. Noise;

3. Uneven illumination;
4. Presence of artefacts.

Several techniques can be applied to deal with these problems. The images have to be pre-processed so as to produce a better quality image for assuring that only relevant information is present in the images. Here, classical solutions for image enhancement, de-noising etc are discussed and compared.

### Noise Removal

The goal of this process is to reduce noise or useless details without introducing too much alterations or distortions to the original image.

In [1] salt and pepper noise which is random black and white pixels on the image is removed by using Gaussian blur which is expressed using the equation:

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

where  $r$  denotes the blur radius ( $r^2 = x^2 + y^2$ ),  $\sigma$  is the standard deviation of Gaussian distribution. The values of this function are used to create a matrix which is convolved with the image.

However, in practice one finds that this technique blurs the edges of the image along with blurring the noise. Thus it is not desirable to use this approach for vein images.

A common solution to salt and pepper noise is Median filter which replaces the value of a pixel by the median of the grey levels in the neighbourhood of that pixel. In [2] median filter is mathematically expressed using the formula:

$$F(x, y) = \text{median} \{g(s, t)\} \text{ where } (s, t) \in S_{xy}, \text{ the window size.}$$

Advantage of median filter is that it preserves the edges unlike other de-noising techniques. Thus it is highly used in digital image processing.

### Enhancement

Image enhancement is the process of improving the quality of the image. Since vein images are usually of poor quality enhancement is of vital importance. There are lot of image processing algorithms as well as other special methods proposed particularly for enhancing vein images. This section describes some such enhancement techniques.

In [3] image negative, a simple contrast enhancement operation which converts each input pixel into a new value is used to enhance hand vein images. It enhances white or grey details of an image more. The negative of an image can be obtained using the expression:

$$I_{\text{NEG}}(x, y) = 255 - i(x, y).$$

However, this operation also enhances highly illuminated regions which is not a desirable result for accurate vein detection.

In [2, 3], histogram equalization (HE) is used which is a contrast adjustment technique making use of histograms. Histogram is a graphical representation of grey value distribution of an image by plotting the number of pixels for each grey value.

If a grey scale image  $X = x(i, j)$ , with  $n$  pixels distributed among  $L$  discrete intensity levels and  $n_k$  is the total number of pixels with same intensity. Then, probability density function and cumulative density function is calculated as follows [2]:

$$\text{pdf}(X) = \frac{n_k}{n} \text{ where } 0 \leq k \leq (L-1);$$

$$\text{cdf}(X) = \sum_{i=0}^k \text{pdf}(X_i).$$

HE increases contrast but it may also introduce undesirable patterns or noises.

Elliptic high pass filters are used to enhance small blurred finger vein textures followed by histogram equalisation [3]. The transfer function is given as:

$$H(\Omega) = a + b \left( 1 - \frac{1}{\sqrt{1 + \varepsilon^2 F_n^2(\omega)}} \right)$$

where  $\Omega = j\omega$ ;  $F_n$  is the  $n$ th order elliptical polynomial;  $\varepsilon$  determines the pass-band ripple attenuation at the cut off frequency. 'a' is an offset and 'b' is a constant greater than 1.

Contrast limited adaptive histogram equalization (CLAHE) overcomes the problems of histogram equalization such as over enhancement of darker or brighter images and increases visibility of unwanted noise [4, 5]. Contrast limited adaptive histogram equalization can be expressed using:

$$g = g_{\min} - \left(\frac{1}{\alpha}\right) * \ln [ 1 - P(f) ], \text{ where}$$

$g_{\min}$  is the minimum pixel value,

$\alpha$  is the clip limit

and  $P(f)$  is the cumulative density distribution.

Even though CLAHE produces results better than HE, it also introduces noise sometimes.

Usually vein appear as dark ridges due to NIR imaging modes. Gabor filter can be used to enhance vein images. A Gabor filter is defined using a Gaussian function modulated by a complex sinusoidal signal [6,7]. The Gaussian function is given by:

$$G(x, y) = \rho \exp \left\{ -\frac{1}{2} \left( \frac{x_{\theta}^2}{\sigma_x^2} + \frac{y_{\theta}^2}{\sigma_y^2} \right) \right\} \exp (j2\pi f_0 x_{\theta} )$$

where,

$$\begin{bmatrix} x_{\theta} \\ y_{\theta} \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_{\theta} \\ y_{\theta} \end{bmatrix}$$

$\rho = 1 / (2\pi\sigma_x\sigma_y)$ ,  $\hat{j} = \sqrt{-1}$ ,  $\theta$  is the orientation of a Gabor filter,  $f_0$  denotes the filter central frequency,  $\sigma_x$  and  $\sigma_y$  are the two axes of Gaussian envelop,  $x_{\theta}$  and  $y_{\theta}$  are rotated version of coordinates  $(x, y)$  of Gabor filter.

In [4] the even part of Gabor filter is used to obtain the vein ridges clearly. The filtered vein image  $r(x, y)$  is obtained by convolving an input image  $I(x, y)$  with the even symmetric Gabor filter  $G_k^e(x, y)$  at the  $k$ th channel. That is

$$r_k(x, y) = G_k^e(x, y) * I(x, y),$$

where '\*' represents two dimensional convolution operation.

A Gabor filter better enhances an image in comparison with other techniques. However, Gabor filters also suffers from false information introduction which is due to the characteristics of Gabor function.

## FEATURE EXTRACTION

This process focuses on extracting a set of discriminatory features from the acquired biometric data for further recognition. There exist many methods to extract such features from finger vein images. Feature extraction is a major step in recognition process.

### Mathematical Morphology

Mathematical morphology has a place in feature extraction process as it emphasizes in shape information. It is considered as a powerful tool to extract information from images based on set theory, random functions, topology etc.

#### *Erosion and Dilation*

These are the primary morphological operation. Erosion shrinks or thins the object in an image where as Dilation grows the objects.

Erosion of a binary image  $f$  by a structuring element  $s$ , which is a small binary image with specific shape, size and origin, denoted as  $g=f \ominus s$  produces a new binary image  $g$ . Larger structuring element have more effect. If  $s_1$  and  $s_2$  are a pair of structuring elements which are identical in shape, with  $s_2$  twice the size of  $s_1$  then :

$$f \ominus s_2 \approx (f \ominus s_1) \ominus s_1$$

Erosion removes small-scale details from a binary image but, reduces the size of the region of interest.

The dilation of a binary image  $f$  by a structuring element (SE)  $s$  produces a new binary image  $g=f \oplus s$ . It has opposite effect of erosion. It adds a layer of pixels to inner and the outer boundaries of the regions. The result of these methods is influenced by the size and shape of the structuring element. Erosion and dilation are opposite to each other and thus are referred to as dual operations. Formally, duality is written as:

$$f \oplus s = f^c \ominus s_{rot}$$

where,  $s_{rot}$  is the SE rotated 180°

#### *Opening and Closing*

These are two elementary combinations of erosion and dilation. Opening smoothes contours and eliminate islands, where as closing fills narrow gulfs, and eliminates holes.

Opening (erosion followed by dilation) is denoted as,  $(f \circ s) = (f \ominus s) \oplus s$  where  $f$  denotes the image and  $s$  the structuring element. Closing (dilation followed by erosion) is denoted as,  $(f \bullet s) = (f \oplus s) \ominus s$  where  $f$  denotes the image and  $s$  the structuring element.

### ***Morphological Hit or Miss Transform***

It finds the occurrences of an object and its surroundings in an image. It is used to select certain points with particular geometric properties to perform matching.

The operation requires a matched pair of SE,  $\{s_1, s_2\}$ . Hit or Miss Transform is expressed as:

$$f \otimes \{s_1, s_2\} = (f \ominus s_1) \cap (f^c \ominus s_2)$$

### ***Minutiae***

Many finger vein systems are based on minutiae. Minutia based techniques represents a vein pattern by its local features like end points and bifurcations [8,9]. Before extracting minutiae from a given image, the above mentioned morphological methods are applied for binarization and thinning.

### ***Crossing Number***

To extract minutiae, crossing number method is employed in [8]. A pixel wise operation is performed to obtain the bifurcation points and end points. For a 3x3 region  $P_0, P_1, \dots, P_8$  with  $P_0$  as centre pixel, the following equation is used to find whether  $P_0$  is a branching point or not. A similar approach is used to find the end points. Minutiae points can be found out using the equation:

$$CN = 0.5 \sum_{i=1}^8 |P_{i+1} - P_i|, \text{ where } P_9 = P_1$$

However, these techniques may introduce a lot of spurious minutiae along with true ones and needs a lot of post processing especially when dealing with bad quality vein images. This adversely affects the decision stage. In [10] Morphological Hit or Miss based minutiae extraction is employed. This method avoids post processing effort as it will not produces any false minutiae.

### ***Principle Component Analysis (PCA)***

In [11] 2D<sup>2</sup>PCA is used for extracting features from vein images. PCA is a linear dimensionality reduction and feature extraction method. To deal with 2D images a 2DPCA and 2D<sup>2</sup>PCA are used. The process is as follows:

Let the M finger vein images considered, are denoted by  $A_1, \dots, A_M$ , The mean image matrix is calculated as,  $\bar{A} = (1/M) \sum_j A_j$  and the image covariance matrix is calculated as,

$$G = \frac{1}{M} \sum_{j=1}^M (A_j - \bar{A})^T (A_j - \bar{A}).$$

For a random image matrix A, the key of obtaining new features is to get a projection matrix  $X \in R^{n \times d}$ ,  $n \geq d$ . Then new features are calculated as,  $Y = AX$ .

However, although PCA extracts global features, it may ignore some local information details.

### ***Local Binary Pattern (LBP)***

In [12] LBP is used as a feature extraction method. An LBP operator is a 3x3 kernel for texture classification. LBP is mathematically obtained using the formula:

$$LBP(x_c, y_c) = \sum_{n=0}^{n=7} s(i_n - i_c) 2^n$$

where,

$$s(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

$i_c$  and  $i_n$ : grey scale values of center pixel ( $x_c, y_c$ ) and its 8 neighbouring pixels.

LBP is not a good method as it can only represent only a part of a finger vein image and it could not represent delicate differences among the grey values of pixels.

## MATCHING

The matching process is the last step in any biometric recognition system. In this process the query image is compared with the stored image or features to decide whether the user is authorized or not. Depending on the type of the feature extraction methods there are a variety of matching techniques. Matching process is regarded as a measure of similarity.

Researchers who have used PCA for feature extraction mostly used Euclidean distance to compute the matching score. On a successful match it is expected that the distance is zero or as low as possible.

### Euclidean Distance

It can be used to calculate the absolute distance between the feature points and the given points. It is applied in [13] for finding the distances between all the minutiae and core points. Euclidean distance is calculated as:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### Correlation Based

This approach is used to evaluate the similarity degree of a reference subset with another subset based on some criteria. This simple principle is used for matching in [14, 15].

Let  $f(m, n)$  and  $g(m, n)$  be two images and  $F(m, n)$ ,  $G(m, n)$  be the 2D Fourier Transform of the images respectively. Then,

$$F(u, v) = \sum_{m=-M}^M \sum_{n=-N}^N f(m, n) e^{-\frac{2jum\pi}{M}} e^{-\frac{2jun\pi}{N}}$$

$G(u, v)$  can be obtained in the same way. The cross phase spectrum of the two images are given by,

$$R_{FG}(u, v) = F(u, v) \overline{G(u, v)}$$

where,  $\overline{G(u, v)}$  is the complex conjugate of  $G(u, v)$ .

The conjugate function is the 2D inverse DFT of  $R_{FG}(u, v)$

$$r_{fg} = \frac{1}{MN} \sum_{u=-M}^M \sum_{v=-N}^N R_{fg}(u, v) e^{\frac{2jum\pi}{M}} e^{\frac{2jun\pi}{N}}$$

### Phase Only Correlation

In [16] another method called POC is used. It is similar to correlation except the difference in calculation of cross phase spectrum. This can be expressed using:

$$\text{ReFG}(u, v) = \frac{F(u, v)\overline{G(u, v)}}{|F(u, v)\overline{G(u, v)}|}$$

Correlation based methods are simple solutions to the matching problem and they produce less number of errors. However, this method demands a high computational power which makes it less applicable for real time applications.

### Hausdorff Distance Method

This method proposed in [17,18] compares the similarity of two sets rather than comparing the images. This is commonly used when the features extracted are represented as a set of points like minutiae sets. For two point sets  $X = \{x_1, x_2, x_3, \dots\}$  and  $Y = \{y_1, y_2, y_3, \dots\}$  the Hausdorff distance is computed as:

$$\text{HD}(x, y) = \max(d(X, Y), d(Y, X))$$

$$d(X, Y) = \max_{x_i \in X} \min_{y_j \in Y} \|x_i - y_j\|$$

The smaller the value of HD, the more similar the points are.

### Modified Hausdorff Distance Method

However, Hausdorff distance method is sensitive to small changes in point locations. To overcome the problems with Hausdorff method, a modified method has been introduced in [19,20]. The modified Hausdorff distance is defined as:

$$\text{HD}(x, y) = \max(d(X, Y), d(Y, X))$$

$$d(x, y) = \frac{1}{N} \sum_{x_i \in X} \min_{x_j \in Y} \|x_i - y_j\|$$

### CONCLUSION

In this paper, we have sketched some of the fundamental mathematical concepts in finger vein biometrics. We have analysed and compared the existing mechanisms in each step of the finger vein processing. It is important to note that none of these steps have been satisfactorily addressed yet, and all the algorithms and mathematical techniques described have room for further improvement. In the pre-processing step, none of the enhancement methods were appropriate for vein enhancement without introducing noise or artefact. Feature extraction methods also had weaknesses. Finally, the matching method has to be carefully chosen according to the feature extraction method. Finding new mathematical methods incorporated with existing image processing methods can lead to novel research results. We can conclude that mathematical techniques have a promising impact on finger vein image processing and thus a better mathematical method means a better finger vein biometric system.

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