ISSN: 2395-6607, Vol. 1, No. 1 March 2015, pp. 28–34

MATCHING PRECLUSION NUMBER OF RADIX TRIANGULAR MESH

D. Antony Xavier[1] and Deeni C.J.[2]
Department of Mathematics, Loyola College, Chennai–34
E-mail: [2]srdeenicj@gmail.com

Abstract—*In this paper we use the concepts of matching,perfect matching, matching preclusion number, and conditional matching preclusion number. A radix triangular mesh denoted as Tn, consists of a set of vertices V(Tn) = { (x,y) / 0 x+y n}. Where any two vertices (x1,y1) and (x2,y2) are connected by an edge if x1 - x2 + y1 - y2 < n-1. In this paper we find out the values of mp (Tn) when n(n+1) 0(mod 4). Minimum matching preclusion in radix triangular mesh is trivial.*

Keywords*: Matching Preclusion Number, Matching, Perfect Matching, Radix Triangular Mesh*

INTRODUCTION

Let G be a graph of order n, and also consider this n is even. A matching M of G is a set of pairwise non-adjacent edges. A perfect matching in G is a set of edges such that every vertex is incident with exactly one edge in this set. The matching preclusion number of graph G, denoted by mp (G), is the minimum number of edges whose deletion leaves the resulting graph without a perfect matching. Mp (G) = 0 if G has no perfect matching's. The concept of matching preclusion was introduced by Birgham et.al [7] and further studied by Cheng and Liptak [1, 2] with special attention given to interconnection networks. In [2] Park also put forward some results on this perfect matching. In [7], the matching preclusion number was determined for three classes of graphs, namely, the complete graphs, the complete bipartite graphs $K_{n,n}$ and the hypercube. Hypercube are classical in the area of interconnection networks, and have generated a considerable amount or research including fault tolerant routings, strong connectivity properties, various Hamiltonian properties and some others also. In certain applications, every vertex requires a special partner at any given time and the matching preclusion number measures the robustness of this requirement in the event of edge failures as indicated in [7]. Hence in these interconnection networks, it is desirable to have the property that the only optimal matching preclusion sets are those whose elements are incident to a single vertex. The following propositions are obvious.

PROPOSITION

Let G be a graph with an even number of vertices. Then mp(G) ≤ δ(G), where, δ(G) is the minimum degree of G.

In [4] E. Cheng et.al proved somany results related to k-regular bipartite graph. If we gone through the literature survey [2, 5, 8] the authors proved various results related to the perfect matching preclusion number and conditional matching preclusion number of different types of graphs and networks.

In the next section we find out the matching preclusion number of radix triangular mesh.

MATCHING PRECLUSION FOR RADIX N TRIANGULAR MESH

Definition [6] A radix n-triangular mesh network, denoted as T_n, consists of a set of vertices V (Tn) ={ (x,y)/ $0 \le x + y \le n$} where any two vertices (x_1,y_1) and (x_2,y_2) are connected by an edge if and only if $|x_1-x_2|+|y_1-y_2| = n-1$. The number of vertices and edges in T_n is equal to n (n+1)/2 and 3n(n-1)/2 respectively.

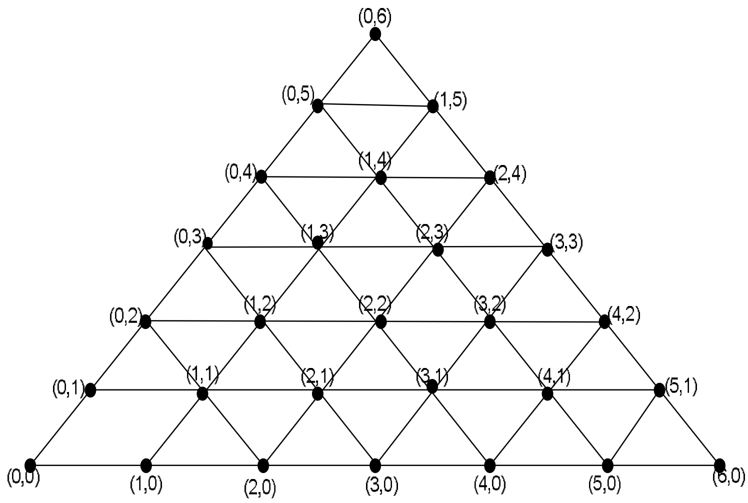

Fig. 2.1: Radix Triangular Mesh T7

Lemma 2.1 [6] Any triangular mesh network T_n is Hamiltonian. (See figure 2.2)

Lemma 2.2 Let n be an integer and $n(n+1) \equiv 0(\bmod 4)$. Then T_n has exactly two edge disjoint perfect matching's.

Proof:

Alternative edges in a Hamiltonian cycle will form two edge disjoint perfect matching M_1 and M_2.

Let the edge (0,0) and (1,0) $\in M_1$, Then (0,0) and (0,1) $\in M_2$.. Suppose M_3 is another perfect matching in T_n. Then M_3 contains either (0,0), (1,0) or (0,0),(0,1).This implies that M_3 is not edge disjoint perfect matching from M_1 and M_2.

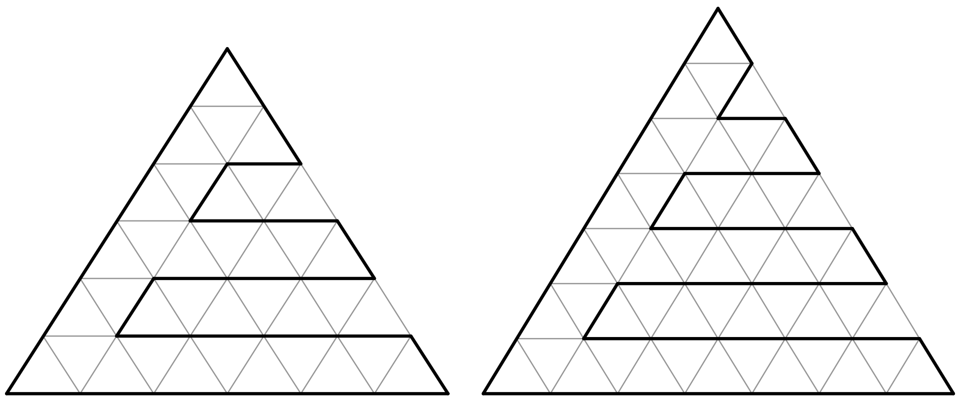

Fig. 2.2: Hamiltonian Cycle in Tn

Theorem 2.1

Let n> 3 be an integer and $n(n+1) \equiv 0(\bmod 4)$. Then $mp(T_n) = 2$

Proof:

By Lemma 2.2,T_n has two edge disjoint perfect matching's M_1 and M_2.

$\therefore mp(T_n) > 1$

But $mp(T_n) \leq \delta = 2$ by prop 1.1

$\therefore mp(T_n) = 2$

Theorem 2.2

Let n> 3 be an integer and $n(n+1) \equiv 0 (\bmod 4)$. Then every minimum matching preclusion set in T_n is trivial.

Proof

According to Lemma 2.2, T_n has two edge disjoint perfect matching's and let it be $M_1$and M_2. Assume that, the edges r = (0,0),(1,0) ∈ M_1 and s = (0,0),(0,1) ∈ M_2. By Theorem 2.1, mp (T_n) = 2.

Let F be the preclusion set in T_n and $|F| = 2$. Let F = $\{x, y\}$

Case (1) Let $x, y \in M_1$.

Then $M_2 \subseteq T_n - F$

Similarly $x,y \in M_2$.

Case (2) $x \in M_1$ and $y \in T_n - \{M_1, M_2\}$

Then $M_2 \subseteq T_n - F$.

Case (3) Let $x = s \in M_2$, $y \in M_1$ and $y \neq r$,y is an interior edge.

Suppose y is an interior edge of T_n. Then y is a side of a parallelogram.

Clearly opposite side of the parallelogram is also in M_1.

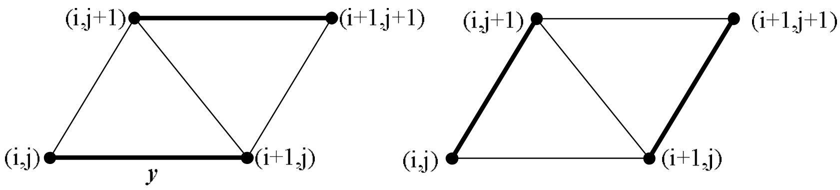

Fig. 2.3

Let y = (i,j),(i+1,j) ∈ M_1 and clearly the opposite side of the parallelogram is also in M_1i,e. (i,j+1),(i+1,j+1) ∈ M_1. Now M_1 + [(i,j)(i,j+1), (i+1,j)(i+1,j+1)]– [y, (i,j+1)(i+1,j+1)] is a perfect matching in T_n – F.

Case(4) Let $x = s \in M_2$, $y \in M_1$ and $y \neq r$,y is a boundary edge.

Let y = (i,j),(i,j+1)

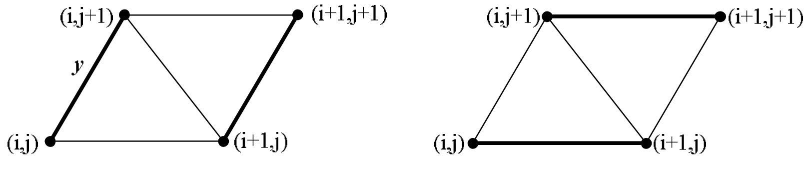

Fig. 2.4

Similar as Case (3).

(b) Let *y = (i+2,j),(i+1,j+1)*

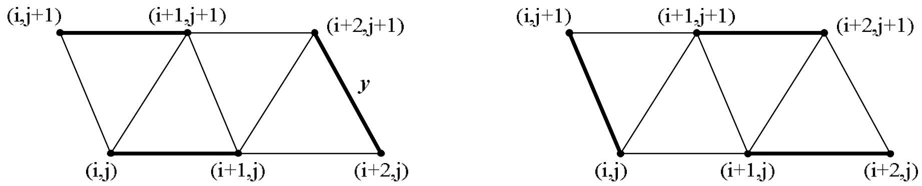

Fig. 2.5

Then (i,j), (i+1,j), (i-1,j+1), (i,j+1) ∈ M_1. Now M_1 + [(i,j)(i-1,j+1), (i,j+1)(i+1,j+1), (i+1,j)(i+2,j)]–[y, (i,j)(i+1,j), (i-1,j+1)(i,j+1)] is a perfect matching in T_n–F.

(c) Let y = (i,j),(i,j+1)

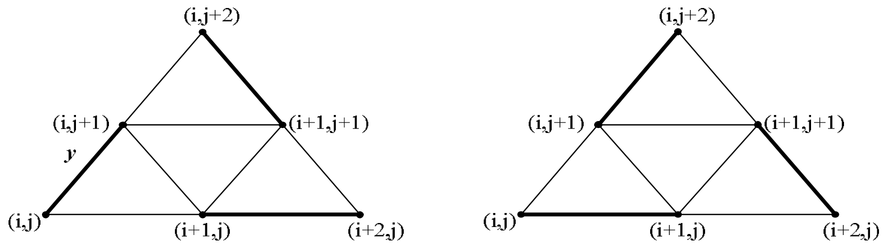

Fig. 2.6

Then (i+1,j),(i+2,j), (i+1,j+1),(i,j+2) ∈ M_1. Now M_1 + [(i,j)(i+1,j), (i,j+1)(i,j+2),(i+2,j)(i+1,j+1)]–[y, (i+1,j)(i+2,j),(i+1,j+1)(i,j+2)] is a perfect matching inT_n – F.

Case (5) Let x = s ∈ M_2, y ∈M_1 and y ≠ r,y is a corner boundary edge.

Let y = (i+1,j+1),(i,j+2)

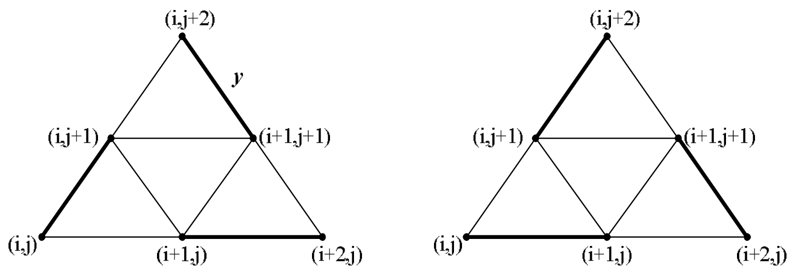

Fig. 2.7

Then (i+1,j),(i+2,j), (i,j),(i,j+1) ∈ M_1. Now M_1 + [(i,j+1)(i,j+2), (i,j)(i+1,j),(i+1,j+1)(i+2,j)]–[y, (i,j)(i,j+1),(i+1,j)(i+2,j)] is a perfect matching in T_n – F.

Case (6) Let x ≠ s ∈ M_2, y ∈M_1 and y ≠ r,y both are adjacent and interior edges.

(a) Letx= (i+1,j),(i+1,j+1) ∈M_2, y = (i+1,j+1),(i+2,j+1) ∈M_1.

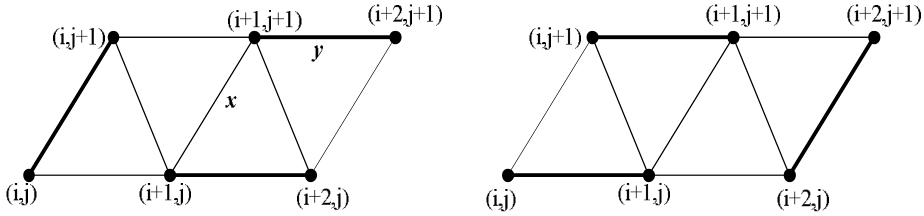

Fig. 2.8

Then (i,j),(i,j+1), (i+1,j),(i+2,j) ∈ $M_{1.}$ Now M_1 + [(i,j)(i+1,j), (i,j+1)(i+1,j+1),(i+2,j)(i+2,j+1)]–[y, (i,j)(i,j+1),(i+1,j)(i+2,j)] is a perfect matching in T_n – F.

(b) Letx = (i,j+1),(i+1,j+1) $\in M_2$, y = (i+1,j+1),(i+2,j+1) $\in M_1$.

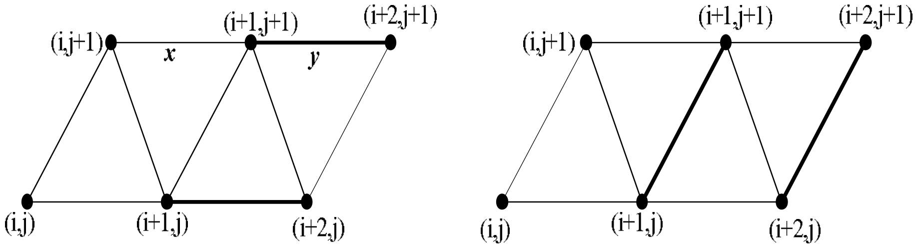

Fig. 2.9

Then (i+1,j),(i+2,j)∈ $M_{1.}$ Now M_1 + [(i+1,j)(i+1,j+1), (i+2,j)(i+2,j+1)]–[y, (i+1,j)(i+2,j)] is a perfect matching in T_n – F.

Case (7) Let x ≠ s ∈ M_2, y $\in M_1$ and y ≠ r,y both are adjacent and any one is interior edge.

(a) Letx = (i,j+2),(i+1,j+2) $\in M_2$, y = (i+2,j+1),(i+1,j+2) $\in M_1$.

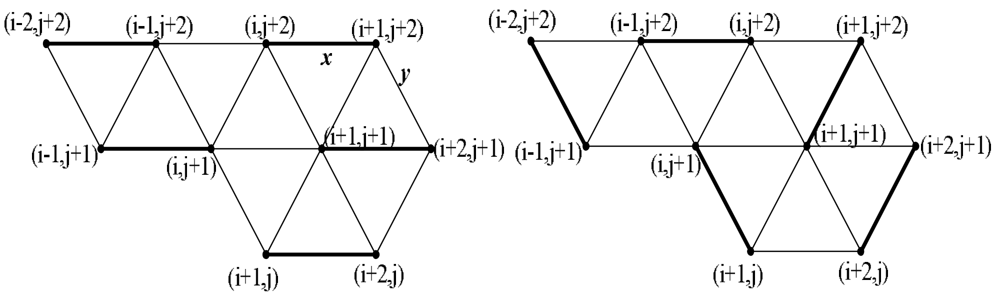

Fig. 2.11

Then (i-2,j+2)(i-1,j+2), (i-1,j+1)(i,j+1),(i+1,j+1)(i+2,j+1), (i+1,j)(i+2,j)∈ $M_2$and

(i-1,j+2)(i,j+2), (i,j+1)(i+1,j+1)$\in M_1$. Now M_2 + [(i-1,j+1)(i-2,j+2), (i-1,j+2)(i,j+2), (i+1,j)(i,j+1), (i+1,j+1), (i+1,j+2), (i+2,j), (i+2,j+1)]–[x, (i-2,j+2)(i-1,j+2), (i-1,j+1)(i,j+1), (i+1,j+1)(i+2,j+1), (i+1,j)(i+2,j)] is a perfect matching in T_n – F.

(b) Letx = (i+1,j+2),(i+1,j+3) $\in M_2$, y = (i+1,j+3),(i,j+4) $\in M_1$.

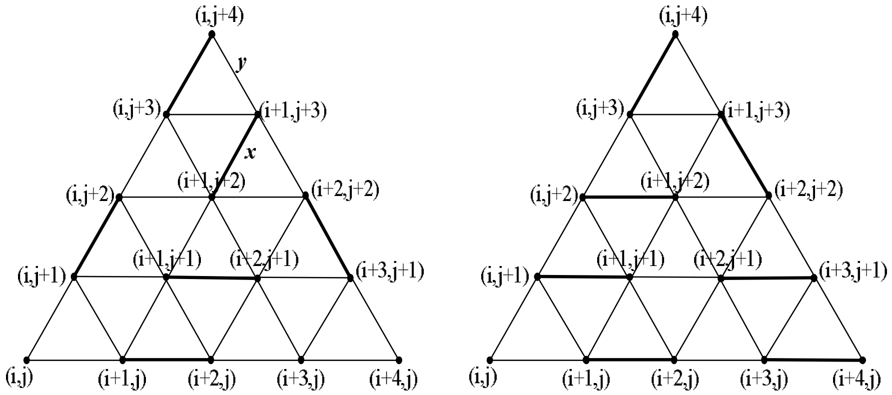

Fig. 2.12

Then (i+1,j)(i+2,j), (i+1,j+1)(i+2,j+1),(i,j+1)(i,j+2), (i+3,j+1)(i+2,j+2),(i,j+3),(i,j+4)∈ M_2. Now M_2 + [(i+1,j)(i+2,j), (i+3,j)(i+4,j), (i,j+1)(i+1,j+1), (i+2,j+1)(i+3,j+1), (i,j+2)(i+1,j+2), (i+2,j+2), (i+1,j+3), (i,j+3), (i,j+4)]–[x, (i+1,j+1)(i+2,j+1), (i,j+1)(i,j+2), (i+3,j+1)(i+2,j+2)] is a perfect matching in T_n – F.

(c) Letx = (i+3,j+1), (i+2,j+2) $\in M_2$, y = (i+1,j+2), (i+2,j+2) $\in M_1$.

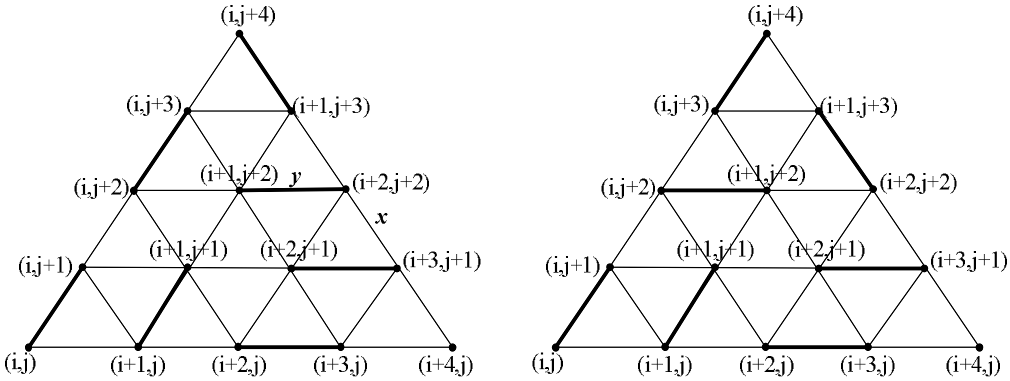

Fig. 2.13

Then (i,j)(i,j+1), (i+1,j)(i+1,j+1), (i+2,j) (i+3,j), (i+2,j+1)(i+3,j+1), (i+1,j+2)(i+2,j+2), (i,j+2),(i,j+3), (i+1,j+3)(i,j+4)∈ M_1. Now M_1 + [(i,j)(i,j+1), (i+1,j)(i+1,j+1), (i+2,j)(i+3,j), (i+2,j+1)(i+3,j+1), (i,j+2)(i+1,j+2), (i+2,j+2)(i+1,j+3), (i,j+3),(i,j+4)] – [y, (i,j+2)(i,j+3), (i+1,j+3)(i,j+4)] is a perfect matching in T_n – F.

ACKNOWLEDGEMENT

This work is supported by Maulana Azad Fellowship F1-17/2013-14/MANF-CHR-KER-30436 of the University Grants Commission, New Delhi, India.

REFERENCES

E. Cheng, L. Liptak, Matching preclusion for some interconnection networks, Networks 50(2007) 173-180

E. Cheng, L. Lensik. M. Lipman, L. Liptak, Matching preclusion for alternating group graphs and their generalizations, Int. J. Found Comp. Sc. 19(2008) 1413- 1437

E. Cheng, L. Lensik. M. Lipman, L. Liptak, Conditional Matching preclusion sets, Inf.Sc. 179(2009) 1092-1101

E. Cheng, P. Hu, R. Jia, L. Liptak, Matching preclusion and conditional matching preclusion for bipartite interconnection networksII, Cayley graphs generated by transposition trees and hyperstars. Networks (2012)

Jung–Heum Park, Matching preclusion problem in restricted HL- graphs and recursive circulantG (2m,4) j.of KISS 35(2)(2008) 60-65

Robert C. Birgham, Frank Harry, ElizebethC. Biolin, JayYellen, Perfect matching preclusion, Congressus Numerantium 174(2005)185-192

H. Sarbazi-Azad, M. Quld- Khaoua, L. Mackenzie, Algorithemic constructions of Hamiltonians in pyramid networks. Inf. proc. letters 80(2001) 75-79

S. Razavi, H. Sarbazi Azad. The triangular pyramid: Routing and topological properties. Inf. Sc. 180(2010) 2328-2339.