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INJECTION/ SUCTION EFFECTS ON AN OSCILLATORY FLOW IN A ROTATING POROUS CHANNEL

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Abstract—*The purpose of the paper is to analyse the effects of injection/suction on an oscillatory flow of a viscous incompressible fluid in a rotating horizontal porous channel. The fluid is injected with constant velocity through the lower stationary plate and is being sucked simultaneously with the same constant velocity through the upper plate oscillating in its own plane about a non-zero constant mean velocity. A closed form solution has been obtained and the effects of injection/suction on the resultant velocities and shear stresses for steady and unsteady flows have been studied.*

Keywords*: Oscillatory, Rotating, Porous Channel, Injection/ Suction*

INTRODUCTION

All fluid phenomena on earth involve rotation to a greater or lesser extent. Those in which rotation is an absolutely essential factor include the large scale circulation in the atmosphere and oceans, and so many other flows with small scale circulation. In the last few years a number of studies have appeared in the literature on rotating flows viz. Vidyanidhu and Nigam (1967) Gupta (1972) Jana and Datta (1977). Injection/ suction effects have also been studied extensively for horizontal porous plate in rotating frame of references by Gupta (1972a) Mazumder(1976) Mazumder *et al.* (1976) Soundalgekar and Pop (1973) for different physical situation. Recently, Mazumder (1991) studied an oscillatory Ekman boundary layer flow bounded by two horizontal flat plates, one of which is oscillating about a non-zero constant mean velocity and the other at rest. An alternative solution to this problem is later given by Ganapathy (1994). By taking hydro magnetic effect and oscillatory fow into account Singh *et al.* (2000, 2005, 2009) further improved the analysis. In the present paper it is proposed to study the injection/ suction effects on the oscillatory flow in a horizontal porous channel in a rotating system.

MATHEMATICAL ANALYSIS

An unsteady flow of a viscous, incompressible fluid is considered between two parallel porous horizontal plates distance d apart. A constant injection velocity, w_0, is applied at the lower stationary plate and the same constant suction velocity, w_0, is applied at the upper plate which is oscillating in its own plane with a velocity $U^*(t^*)$ about a non-zero constant mean velocity U_0. Choose the origin on the lower plate lying in $x^* - y^*$ plane and x^*-axis parallel to the direction of motion of the upper plate. The z^*-axis taken perpendicular to the planes of the plates, is the axis of rotation about which the entire system is rotating with a constant angular velocity Ω^*. Since the plates are infinite in extent, all the physical quantities except the pressure, depend only on z^* and t^*. Denoting the velocity components u^*, v^*, w^* in the x^*, y^*, z^*

directions, respectively, the flow in the rotating system is governed by the following equations:

$$w_z^* = 0, \tag{1}$$

$$u_t^* + w^* u_z^* = -p_x^* / \rho + \upsilon u_{zz}^* + 2\Omega^* v^*, \tag{2}$$

$$v_t^* + w^* v_z^* = -p_y^* / \rho + \upsilon v_{zz}^* - 2\Omega^* u^*, \tag{3}$$

Where, υ is the Kinematic viscosity, t is the time, ρ is the density and p^* is the modified pressure. The boundary conditions for the problem are

$$\left.\begin{aligned} & u^* = v^* = 0, \quad w^* = w_0, \quad at \quad z^* = 0, \\ & u^* = U^*(t^*) = U_0(1 + \varepsilon \cos \omega^* t^*), \\ & v^* = 0, w^* = w_0, at\ z^* = d, \end{aligned}\right\} \tag{4}$$

Where, ω^* is the frequency of oscillations and ε is a very small positive constant.

The integration of the continuity equation (1) under boundary conditions (4) for w^* gives, $w^* = w_0$. Substituting $w^* = w_0$. and eliminating the modified pressure gradient, under the usual boundary layer approximations, equations (2) and (3) reduce to

$$u_t^* + w_0 u_z^* = \upsilon u_{zz}^* + U_t^* + 2\Omega^* v^*, \tag{5}$$

$$v_t^* + w_0 v_z^* = \upsilon v_{zz}^* - 2\Omega^* \left(u^* - U^*\right). \tag{6}$$

Introducing the following non-dimensional quantities

$\eta = z^*/d$, $t = \omega^* t^*$, $u = u^*/U_0$, $v = v^*/U_0$, $\Omega = \Omega^* d^2/\upsilon$ the rotation parameter, $\omega = \omega^* d^2/\upsilon$ the frequency parameter and $s = w_0 d/\upsilon$ the injection/suction parameter, we get

$$\omega u_t + s u_\eta = u_{\eta\eta} + \omega U_t + 2\Omega v, \tag{7}$$

$$\omega v_t + s v_\eta = v_{\eta\eta} - 2\Omega\left(u - U\right), \tag{8}$$

and the corresponding transformed boundary conditions become:

$$\left.\begin{aligned} & u = v = 0 && at \quad \eta = 0, \\ & u = U(t) = 1 + \varepsilon \cos t, \quad v = 0 && at \quad \eta = 1. \end{aligned}\right\} \tag{9}$$

Equation (7) and (8) can now be combined into a single equation, by introducing a complex function $q = u + iv$, as

$$\omega q_t + s q_\eta = q_{\eta\eta} + \omega U_t - 2i\Omega(q - U), \tag{10}$$

and the boundary conditions (9) can also be written in complex notations as:

$$\left.\begin{aligned} q &= 0 && at \quad \eta = 0, \\ q &= U(t) = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right) && at \quad \eta = 1. \end{aligned}\right\} \tag{11}$$

In order to solve equation (10) subject to the boundary conditions (11), we look for a solution of the form

$$q(\eta, t) = q_o(\eta) + \frac{\varepsilon}{2}\left\{q_1(\eta)e^{it} + q_2(\eta)e^{-it}\right\}. \tag{12}$$

Substituting (12) into (10) and (11) and comparing the harmonic and nonharmonic terms, we get

$$q_0'' - sq_0' - l^2 q_0 = -l^2, \tag{13}$$

$$q_1'' - sq_1' - m^2 q_1 = -m^2, \tag{14}$$

$$q_2'' - sq_2' - n^2 q_2 = -n^2, \tag{15}$$

with corresponding transformed boundary conditions

$$\left.\begin{aligned} q_0 = q_1 = q_2 &= 0 \quad at \quad \eta = 0, \\ q_0 = q_1 = q_2 &= 1 \quad at \quad \eta = 1, \end{aligned}\right\} \tag{16}$$

Where, $l^2 = i2\Omega,\ m^2 = i(2\Omega + \omega)$ and $n^2 = i(2\Omega - \omega)$.

The solutions of equations (13) to (15) under the boundary conditions (16) are obtained as:

$$q_0(\eta) = 1 - \left(e^{(r_1 + r_2\eta)} - e^{(r_2 + r_1\eta)}\right) / \left(e^{r_1} - e^{r_2}\right), \tag{17}$$

$$q_1(\eta) = 1 - \left(e^{(r_3 + r_4\eta)} - e^{(r_4 + r_3\eta)}\right) / \left(e^{r_3} - e^{r_4}\right), \tag{18}$$

$$q_2(\eta) = 1 - \left(e^{(r_5 + r_6\eta)} - e^{(r_6 + r_5\eta)}\right) / \left(e^{r_5} - e^{r_6}\right), \tag{19}$$

Where,

$r_1 = \left(s + \sqrt{s^2 + 4l^2}\right)/2$, $r_2 = \left(s - \sqrt{s^2 + 4l^2}\right)/2$, $r_3 = \left(s + \sqrt{s^2 + 4m^2}\right)/2$,

$r_4 = \left(s - \sqrt{s^2 + 4m^2}\right)/2$, $r_5 = \left(s + \sqrt{s^2 + 4n^2}\right)/2$, $r_6 = \left(s - \sqrt{s^2 + 4n^2}\right)/2$,

RESULTS AND DISCUSSION

Now for the resultant velocities and the shear stresses of the steady and unsteady flow, we write,

$$u_0(\eta) + iv_0(\eta) = q_0(\eta) \tag{20}$$

And

$$u_1(\eta) + iv_1(\eta) = q_1(\eta)e^{it} + q_2(\eta)e^{-it} \tag{21}$$

The solution (17) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity components. The amplitude and the phase difference due to these primary and secondary velocities for the steady flow are given by

$$R_0 = \sqrt{u_0^2 + v_0^2}\ ,\ \theta_0 = \tan^{-1}(v_0 / u_0) \tag{22}$$

The amplitude or the resultant velocity R_o and the phase angle θ_0 for the steady part are shown graphically in figure 1.a, b for two cases of rotation parameter Ω small (5, 10) and Ω large (25,50) and the injection/suction parameter *s.* It is observed from figure 1.*a* that for small rotation, Ω, the resultant velocity R_o increases with the increase of the injection/suction parameter *s* (curves I, II) in the whole width of the channel. However, for large rotation and increase in *s* leads to a slight decrease in R_o near the stationary plate and to a slight increase in R_o thereafter in the channel (curves IV, V). This figure also reveals that the amplitude R_o goes on increasing with increasing rotation Ω large or small except that it decreases slightly with the increase in large values of the rotation of the system (curves I, III, IV, VI). For large rotations R_0 rises within a very short distance from the stationary plate to the value unity and oscillates about it. Fig. 1.b shows that phase angle θ_0 for the steady flow increases with increasing injection/suction parameter *s* for any value of rotation large or small. It is also clear that with increasing rotation, Ω, of the system the phase angle θ_0 decreases and becomes approximately zero in the upper half of the channel for large rotation (curve IV, VI). A phase lag is observed for large rotation near the upper oscillating plate.

The amplitude and the phase difference of shear stresses at the stationary plate $(\eta = 0)$ for the steady flow can be obtained as,

$$\tau_{0r} = \sqrt{\tau_{0x}^2 + \tau_{0y}^2}\ ,\ \theta_{or} = \tan^{-1}(\tau_{oy} / \tau_{ox}), \tag{23}$$

Where,

$$\tau_{ox} + i\tau_{oy} = (\partial q / \partial \eta)_{\eta=0} = \left(r_1 e^{r_2} - r_2 e^{r_1}\right) / \left(e^{r_1} - e^{r_2}\right).$$

Here, τ_{ox} and τ_{oy} are, respectively, the shear stresses at the stationary plate due to the primary and secondary velocity components. The numerical values for the resultant shear stress τ_{or} and the phase angle θ_{or}, are listed in table-1. These values in table-1 clearly show that the amplitude τ_{or} of the steady shear stress increases with increasing rotation, Ω, of the system. An increase in the injection/suction parameter, *s,* leads to an increase of τ_{or} for small rotations and to a decrease for large rotations. The phase angle, θ_{or}, decreases with increasing rotation of the system. For any rotation large or small the phase angle, θ_{or}, increases with the increase of injection/suction parameter *s.*

The solutions (18) and (19) together give the unsteady part of the flow. The unsteady primary and secondary velocity components $u_1(\eta)$ and $v_1(\eta)$, respectively, for the fluctuating flow can be obtained as

$$u_1(\eta,t)=\{\mathrm{Re}\,alq_1(\eta)+\mathrm{Re}\,alq_2(\eta)\}\cos t-\{\mathrm{Im}\,q_1(\eta)-\mathrm{Im}\,q_2(\eta)\}\sin t\}, \tag{24}$$

$$v_1(\eta,t)=\{\mathrm{Re}\,alq_1(\eta)-\mathrm{Re}\,alq_2(\eta)\}\sin t+\{\mathrm{Im}\,q_1(\eta)+\mathrm{Im}\,q_2(\eta)\}\cos t\}. \tag{25}$$

The resultant velocity or amplitude and the phase difference of the unsteady flow are given by

$$R_1=\sqrt{u_1^2+v_1^2}\ ,\ \theta_1=\tan^{-1}(v_1/u_1). \tag{26}$$

For the unsteady part, the resultant velocity or the amplitude, R_1 and the phase angle, θ_1, are presented graphically in figure 2.a, b for the two cases of rotation, Ω small (5,10) and Ω large (25,50). Figure 2.a shows that with increase of injection/suction parameter s the resultant velocity R_1, decreases (curves I, II) for small rotations but increases (curves V, VI) for large rotations. For small values of Ω the amplitude R_1, increases (curves I, III) with increase of rotation, however, for large values of rotation Ω, the amplitude R_1, increases (curves V, VII) in the vicinity of the stationary plate but decreases thereafter. Keeping the injection/suction parameter fixed, an increase in the frequency of oscillations ω leads to a decrease of the resultant velocity R_1, (curves I, IV) for small values of rotation but to an increase of R_1, (curves V, VIII) for large values of rotation.

The Fig. 2.b exhibits that the phase angle θ_1, increases (curves I, II**)** with the increase of injection/suction parameter s in whole of the channel width for small rotations, however, for large rotations though it increases (curves V, VI) with s but the phase lead turns into a phase lag near the upper oscillating plate. The phase angle θ_1, decreases with the increase of rotation may it be large (curves V, VII) or small (curves I, III). With the increase of the frequency of oscillations ω the phase angle θ_1, increases except near the oscillating plate where it decreases (curves I, IV) for small rotations. However, for large rotations θ_1 increases (curves V, VIII) with the increase of ω and the phase lead turns into phase lag near the upper oscillating plate where it becomes approximately zero.

For the unsteady part of the flow, the amplitude and the phase difference of shear stresses at the stationary plate (η=0) can be obtained as

$$\tau_{1x}+i\tau_{1y}=(\partial u_1/\partial\eta)_{\eta=0}+i(\partial v_1/\partial\eta)_{\eta=0} \text{ which gives} \tag{27}$$

$$\tau_{1r}=\sqrt{\tau_{1x}^2+\tau_{1y}^2}\ ,\ \theta_{1r}=\tan^{-1}(\tau_{1y}/\tau_{1x}). \tag{28}$$

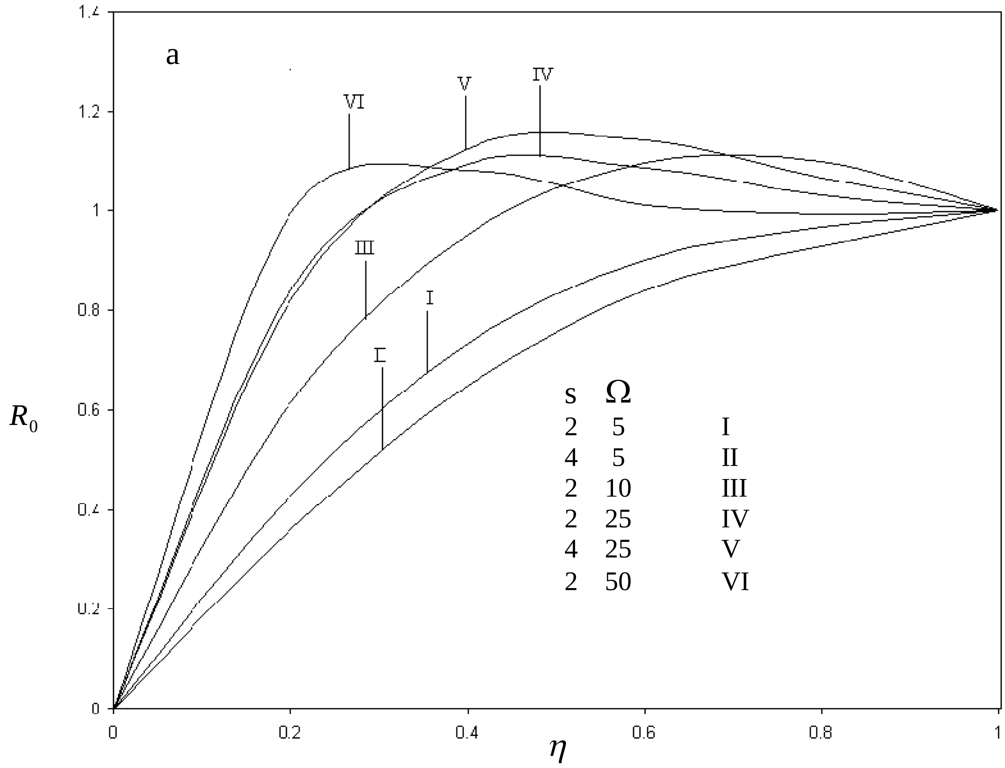
The amplitude τ_{1r} of the unsteady shear stresses are shown graphically in figure 3. This figure clearly shows that the amplitude, τ_{1r}, increases with increasing rotation Ω of the system. For small rotations τ_{1r} first decreases (curves I, II) with the increase of injection/suction parameter s for small frequency of oscillations ω but then increases for large frequency of oscillations. For large rotation a clear cut decrease in τ_{1r} with s is observed in this figure (curves IV, V). The numerical value of the phase difference θ_{1r} are listed in Table 2. It is interesting to note from these values that the phase lead changes to phase lag with the increasing frequency of oscillations ω but at the same time the increasing rotation Ω restricts this effect of ω. For small rotations the effect of injection/suction parameter on the phase lead or lag is insignificant. However, for large rotations the phase lead further increases with increasing injection/suction parameter *s* for all values of frequency of oscillations ω large or small.

Table 1: Values of τ_{or} and θ_{or} for Various *s* and Ω

s	Ω	τ_{or}	θ_{or}
2	5	2.5096	1.0275
2	10	3.8268	0.9419
2	25	6.3956	0.8851
2	50	9.3162	0.8560
4	5	2.9631	1.2121
4	25	5.7741	0.9827

Table 2: Values of θ_{1r} for Various s, Ω and ω

s	Ω	ω					
		0	5	10	15	20	25
2	5	1.028	1.353	1.466	-1.218	-1.092	-1.028
2	10	0.942	1.086	1.244	1.433	1.496	-1.337
2	25	0.885	0.940	0.995	1.051	1.108	1.168
2	50	0.856	0.883	0.910	0.936	0.963	0.991
4	5	1.212	1.566	-1.276	-1.043	-0.889	-0.791
4	25	0.983	1.043	1.102	1.162	1.223	1.286



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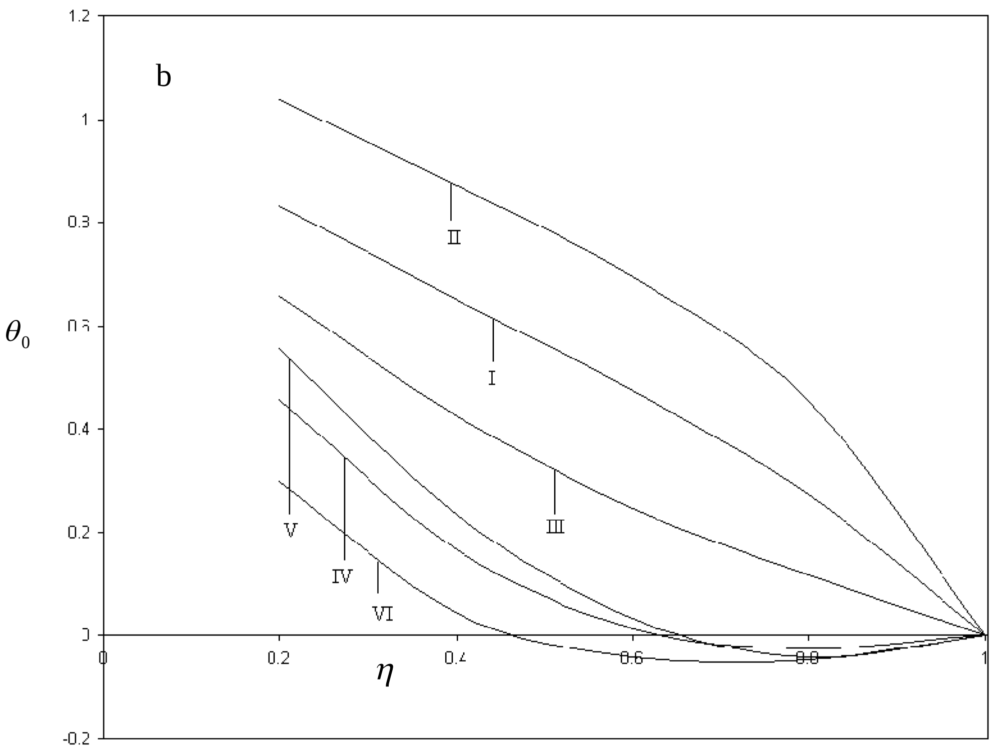
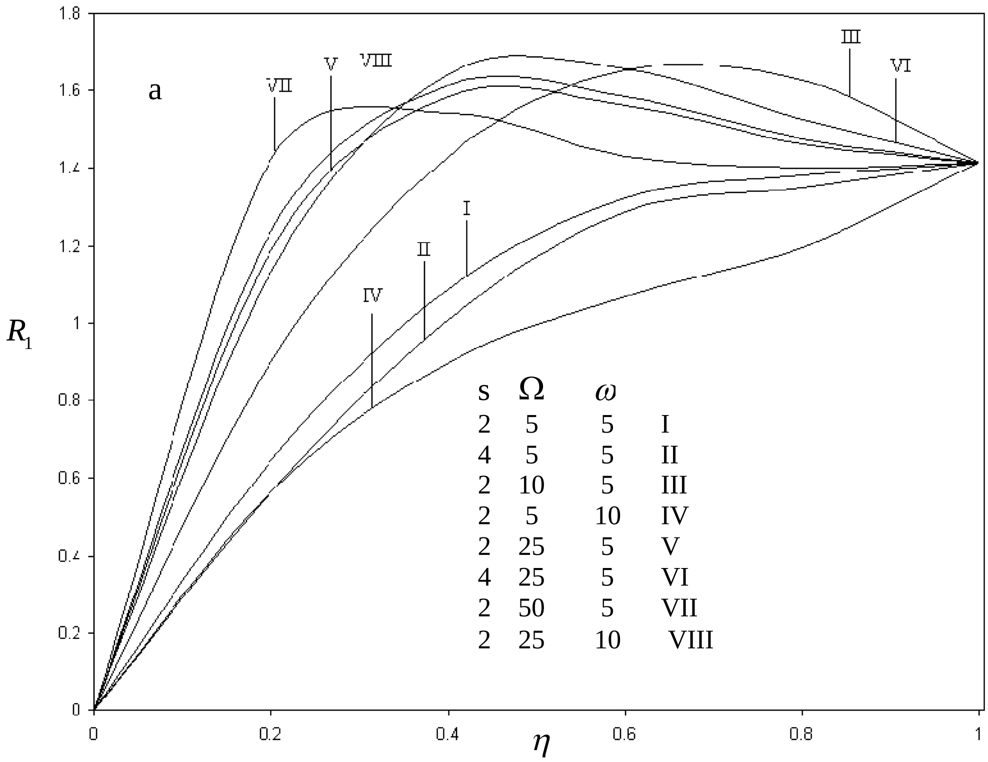


Fig. 1: a,b: Resultant Velocity R_0 and Phase Angle θ_0 Due to u_0 and v_0



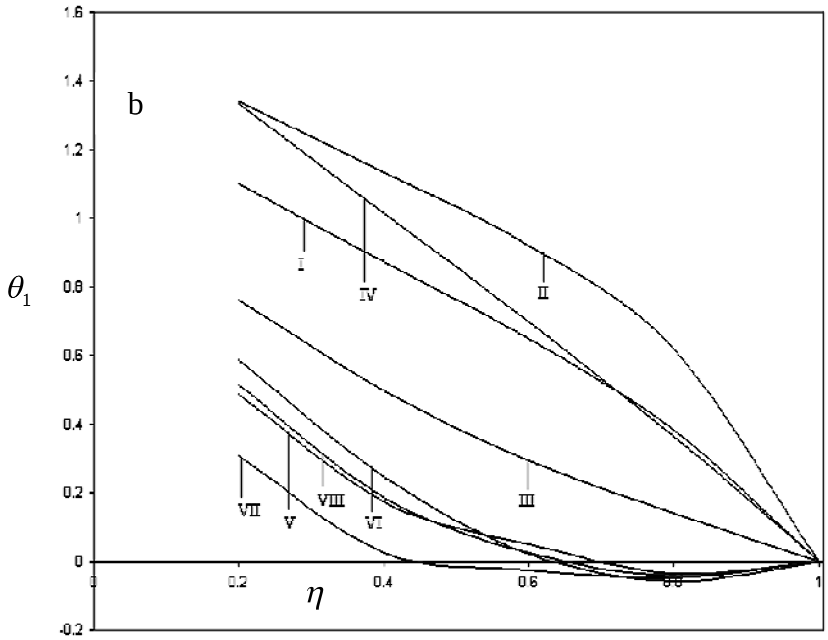


Fig. 2 a,b: Resultant Velocity R_1 and Phase Angle θ_1 Due to u_1 and v_1

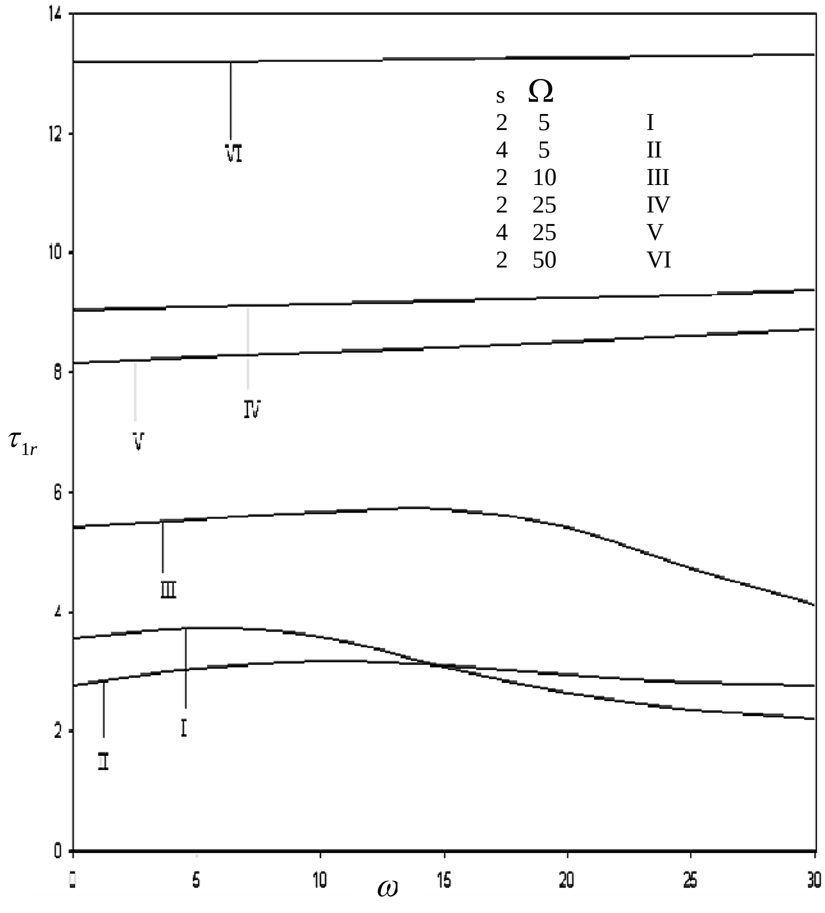


Fig. 3: The Amplitude τ_{1r} of Unsteady Shear Stresses for $t = \pi / 4$

REFERENCES

V. Vidyanidhu and S.D. Nigam, Secondary flow in a rotating channel, J. Math And Phys. Sci. 1, 85, (1967).

A.S. Gupta, Magnetohydrodynamic Ekman layer, Acta Mech. 13, 155-160, (1972).

R.N. Jana and N. Datta, Couette flow and heat transfer in a rotating system, Acta Mech. 26, 301-306, (1977).

A.S. Gupta, Ekman layer on a porous plate, Phys. Fluids 15, 930-931, (1972a).

B.S. Mazumder, Oscillatory hydromagnetic flow of rotating fluid past an infinite plate, Ph.D. Thesis, I.I.T. Khargpur, India.

B.S. Mazumder, A.S. Gupta, and N. Dutta, Flow and heat transfer in the hydromagnetic Ekman layer on a porous plate with hall effects, Int. J. Heat Mass Transfer 19, 523-527, (1976).

V.M. Soundalgekar and I. Pop, On hydromagnetic flow in a rotating fluid past an infinite porous plate, J. Appl. Math. And Mech. (ZAMM), 53, 718-719, (1973).

B.S. Mazumder, An exact solution of oscillatory Couette flow in a rotating system, ASME J. Appl. Mech. 58, 1104-1107, (1991).

R. Ganapathy, A note on oscillatory Couette flow in a rotating system, ASME J. Appl. Mech. 61, 208-209, (1994).

K.D. Singh, An oscillatory hydromagnetic Couette flow in a rotating system, J. Appl. Math And Mech. (ZAMM), 80, 429-432, (2000)

K.D. Singh, M.G. Gorla and Hans Raj. " A Periodic Solution of Oscillatory Couette Flow Through Porous Medium in Rotating System." Indian J. pure appl. Math. 36(3) 151-159, (2005).

K.D. Singh and Alphonsa Mathew. "Injection/Suction Effect on an Oscillatory Hydromagnetic Flow in a Rotating Horizontal Porous Channel." Indian J.Phys. 82(4), 435-445, (2009).